

Exam &  
Key Solutions



**BIRZEIT UNIVERSITY**  
**Electrical Engineering Department**  
**Circuits Analysis- ENEE234**  
**Final Exam (summer 2013-2014)**

Student Name: \_\_\_\_\_

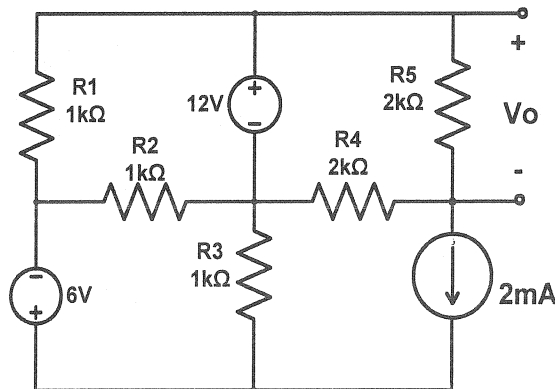
ID: \_\_\_\_\_

**Notes and Instructions:**

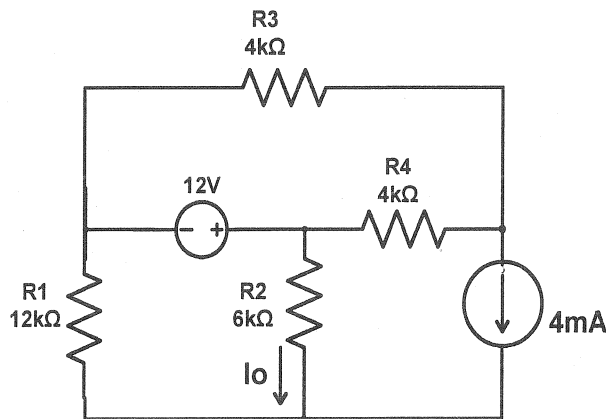
1. Read each question carefully before starting your solution.
3. Write neatly and clearly and leave only one answer for each question.
4. Cell Phones are not allowed.
5. Percentage (%) of total mark is given for each question. Budget your time accordingly

Q1. (25 points)

A. Use superposition to find  $V_o$

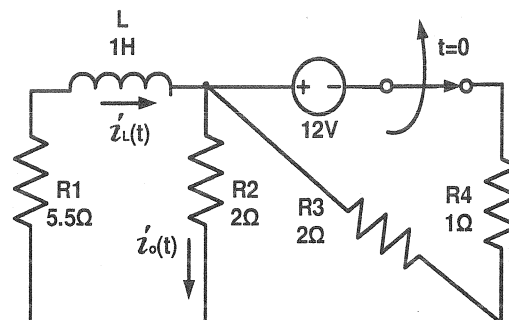


B. Find  $I_o$  using Norton's equivalent circuit



Q2. (10 points)

Find expression for  $i_L(t)$  and  $i_o(t)$



**Q3. (15 points)**

Find the maximum average power that can be transferred to the load  $Z_L$

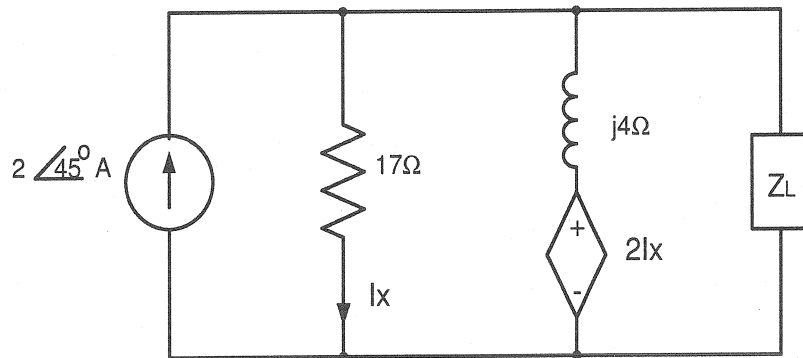


Figure Q3

**Q4. (15 points)**

An Industrial client is charged a penalty if the plant power factor drops below 0.9. The equivalent plant loads are shown in Figure Q4

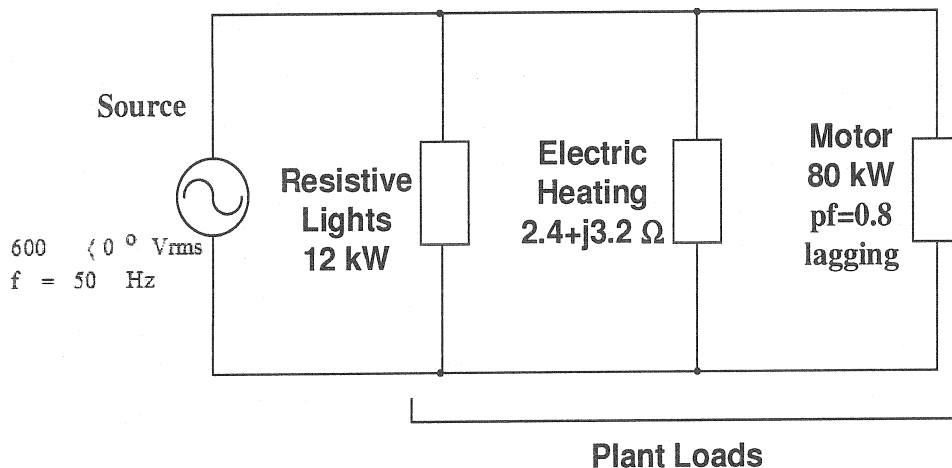
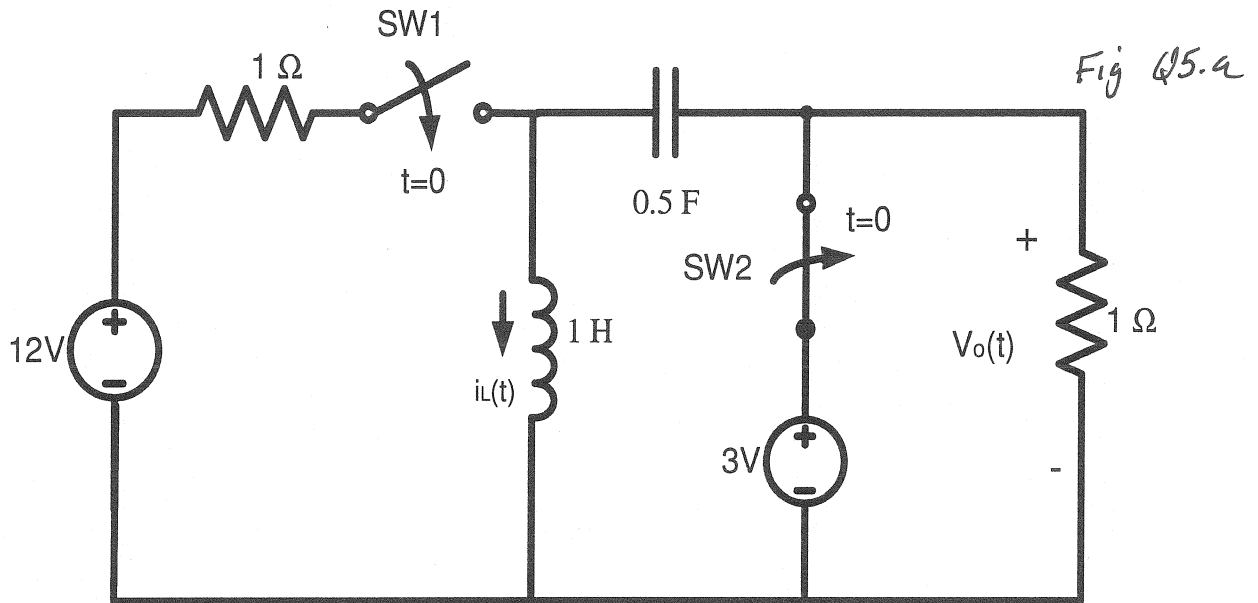


Figure Q4

- Determine total average power ( $P_T$ ), total reactive power ( $Q_T$ ) and total apparent power of the source.
- Determine the value of the required capacitor to be added in parallel with the plant load to raise the plant power factor to 0.9 lagging.
- Find the source current and apparent power for the source with the added capacitor.
- If the plant load was expanded by adding a resistive load of 102kW in parallel to the other loads, is it still necessary to add a capacitor in order to have  $\text{pf} \geq 0.9$  lagging?

## Q5. (20 points)

- a) Use Laplace Transform to find the expression for  $V_o(s)$  for  $t > 0$  in the circuit given in Fig Q5a. Note that at  $t=0$ : SW1 closes and SW2 opens.



- b) Assuming that the expression for  $V_o(s)$  is :

$$V_o(s) = \frac{12(S+3)}{S(S^2+4S+5)}$$

Find  $V_o(t)$

- c) A transfer function  $H(s)$  relating the input voltage  $V_{in}(s)$  to the output voltage  $V_{out}(s)$  is given by the following expression

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{10^3}{S^2 + 2S + 10^3}$$

Find the steady state value of  $V_{out}(t)$  knowing that  $V_{in}(t) = 10 \cos(50t + 15^\circ)$  V

## Q6. (10 points)

Find the h-parameters for the two-port network represented in Fig. Q6

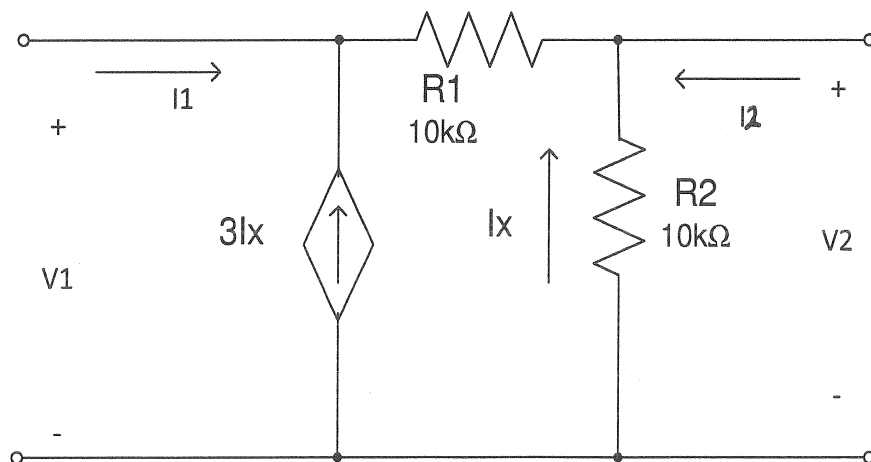
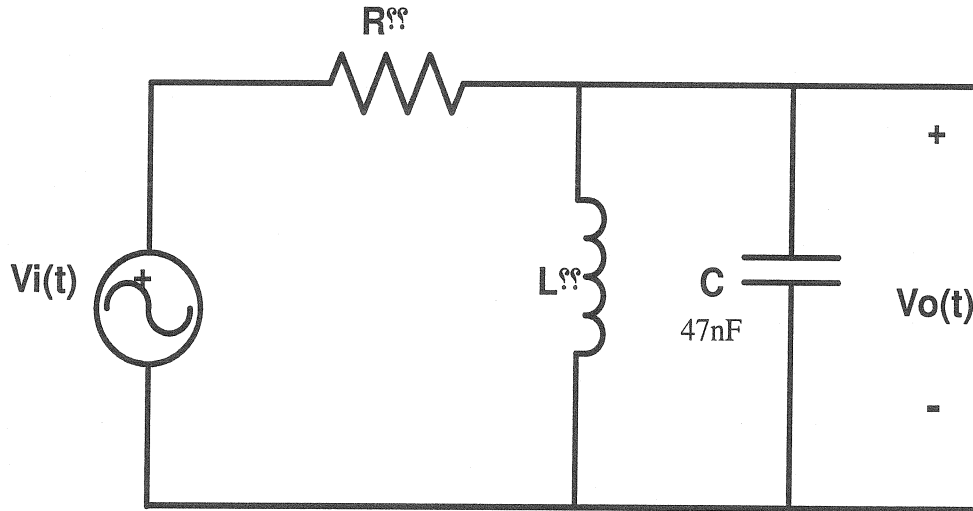


Fig. Q6

## Q7. (10%)

- Show the type of filter which the RLC circuit in Fig.(Q7) is ? (use qualitative analysis)
- Using a 47 nF capacitor , complete the design of the ~~band pass~~ filter, so that its Quality factor  $Q = 10$  and its center frequency  $\omega_0 = 314$  Krad/s.
- Calculate the bandwidth of the filter ( $\beta$ ) .
- Calculate the upper and lower cutoff frequencies ( $\omega_{c1}$  and  $\omega_{c2}$ )
- Assume that the band pass filter of Fig.(Q5) is loaded with a  $1K\Omega$  resistor , calculate the new bandwidth

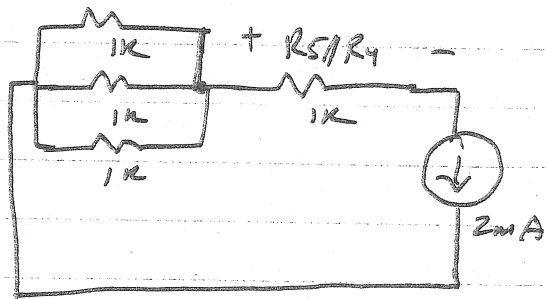
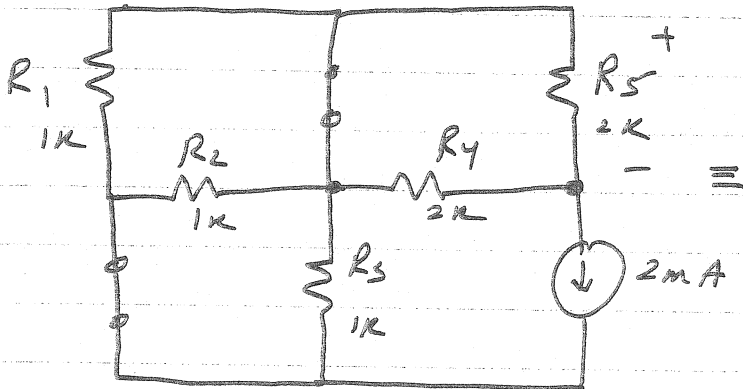


*GOOD LUCK*  
*Nasser Ismail*

Solutions ENEE234  
Final Exam  
Summer 2014

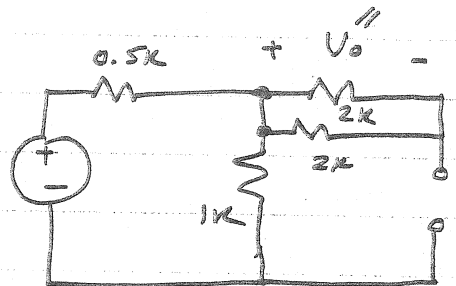
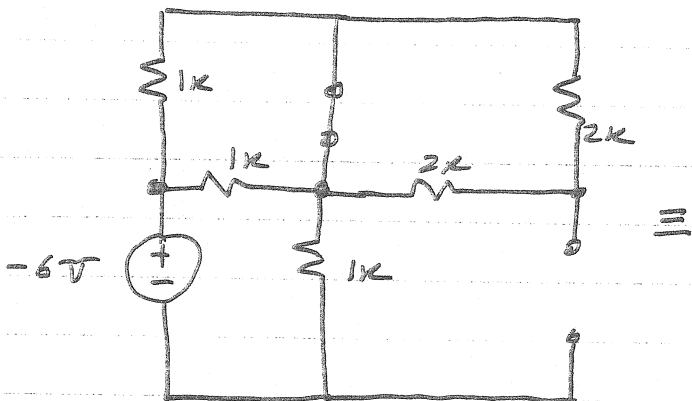
Q1. A. 12 points

4 points



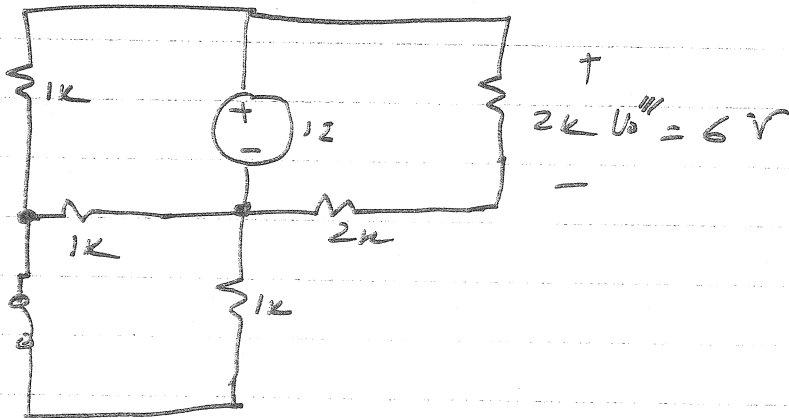
$$V_0' = 2\text{mA} \times 1\text{k}\Omega = 2\text{V}$$

4 points



$$V_0'' = 0\text{V}$$

4 points

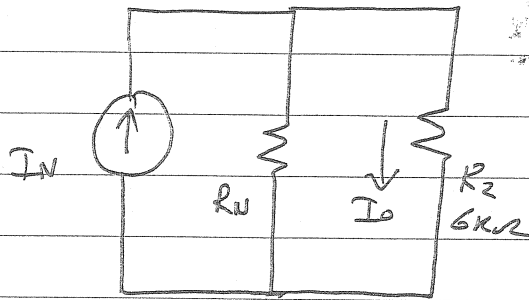


$$V_0''' = 6\text{V}$$

$$V_0 = 0 + 6 + 2 = 8\text{V}$$

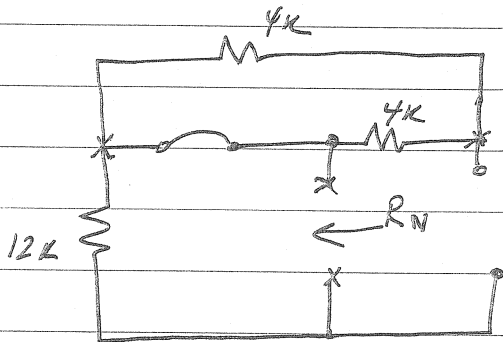
Q1: B

13 points



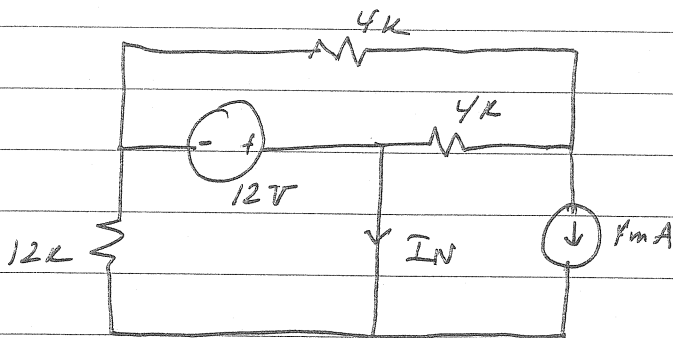
5 points

$$I_0 = I_N \cdot \frac{R_U}{R_U + R_2} = -3 \text{ mA} \times \frac{12}{6+12} = -2 \text{ mA}$$

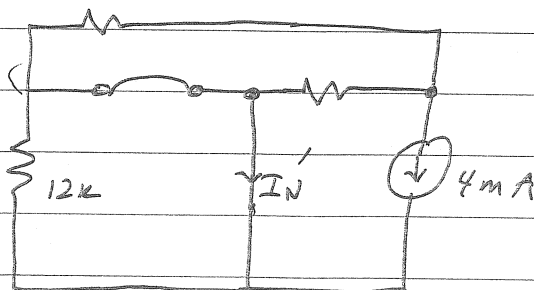


$$R_N = 12 \text{ k}\Omega$$

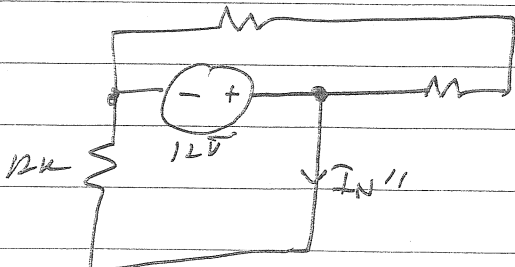
3 points



1)



$$I_N' = -4 \text{ mA}$$



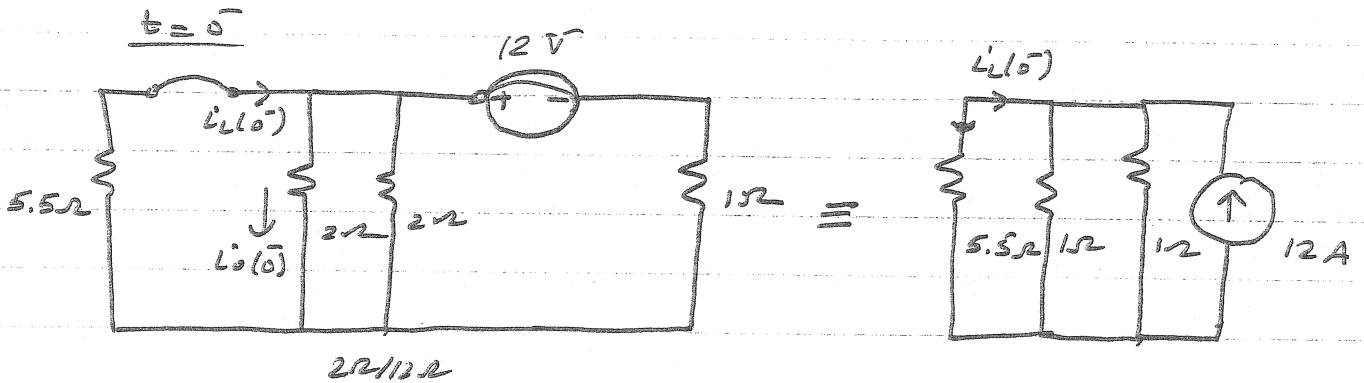
$$I_N'' = \frac{12 \text{ V}}{12 \text{ k}} = 1 \text{ mA}$$

5 points

$$I_N = -4 \text{ mA} + 1 \text{ mA} = -3 \text{ mA}$$

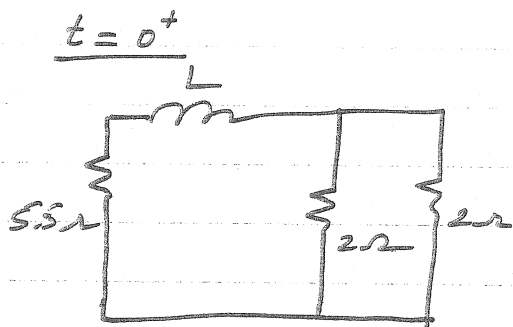
Q2. 10 points

initial conditions



$$i_L(0^-) = -\frac{0.5}{0.5 + 5.5} \times 12 = -1A$$

2 points



$$R_{eq} = (2 // 2) + 5.5 = 6.5\Omega$$

$$\tau = \frac{L}{R_{eq}} = \frac{1}{6.5} = 153.8 \text{ ms}$$

2 points

$t = \infty$

$$i(\infty) = 0$$

2 points

$$i_L(t) = i_L(\infty) + (i_L(0^+) - i_L(\infty)) e^{-t/\tau} \quad A$$

$$i_L(t) = -1 e^{-6.5t} \quad A$$

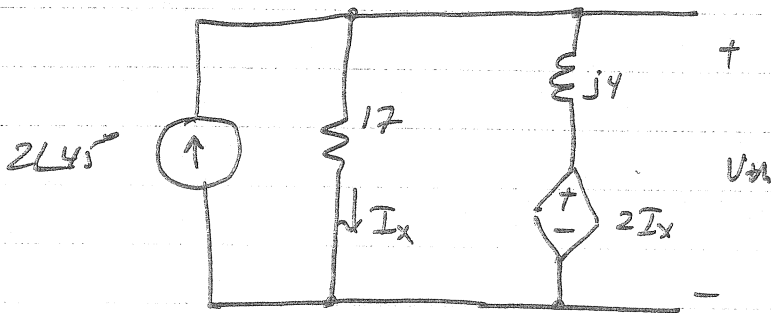
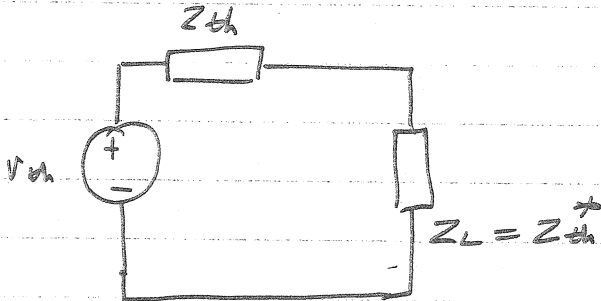
2 points

$$i_o(t) = \frac{i_L(t)}{2} = -0.5 e^{-6.5t} \quad A$$

2 points

Q3.

15 points



$$I_x = \frac{V_{th}}{17}$$

$$2\angle 45^\circ = V_{th} \left( \frac{1}{17} + \frac{1}{j4} \right) - 2I_x \left( \frac{1}{j4} \right)$$

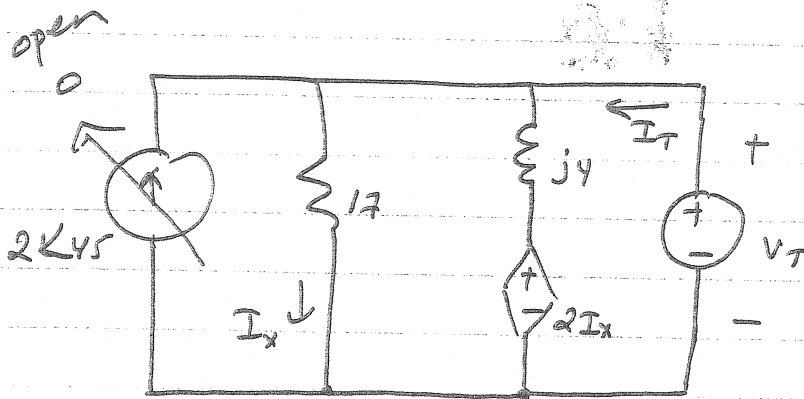
$$2\angle 45^\circ = V_{th} \left( \frac{1}{17} + \frac{1}{j4} \right) - \frac{2V_{th}}{17} \left( \frac{1}{j4} \right)$$

$$2\angle 45^\circ = V_{th} \left( \frac{j4 + 17 - 2}{j68} \right) \Rightarrow \therefore V_{th} = \frac{2\angle 45^\circ \times j68}{15 + j4}$$

$$V_{th} = \frac{136 \angle 135^\circ}{15 + j4} = \frac{136 \angle 135^\circ}{15.52 \angle 14.93^\circ} = 8.76 \angle 120.06^\circ \text{ V}$$

5 points





$$\cancel{2kV} + I_T = V_T \left( \frac{1}{17} + \frac{1}{j4} \right) - \frac{2V_T}{17} \left( \frac{1}{j4} \right)$$

$$I_T = V_T \left( \frac{j4 + 17 - 2}{j68} \right)$$

$$Z_{th} = \frac{V_T}{I_T} = \frac{j68}{15 + j4} = \frac{68 \angle 90}{25.52 \angle 14.93} = 4.38 \angle 75.06^\circ$$

5 points

$$Z_L = Z_{th}^* = 4.38 \angle -75.06 = 1.13 - j4.23$$

2.5 points

$$P_L = \frac{1}{8} \frac{U_{th}^2}{R_L} = \frac{1}{8} (8.76)^2 \cdot \frac{1}{1.13} = 8.475 \text{ W}$$

2.5 points

Q4 15 points

load 1

$$P_1 = 12 \text{ kW}$$

$$\text{pf}_1 = 1 \Rightarrow \theta_1 = 0$$

$$Q_1 = 0$$

$$S_1 = 12 \text{ KVA}$$

load 2

$$S_2 = V_2 I_2^*$$

$$I_2 = \frac{V_2}{Z_2} = \frac{600 \angle 0^\circ}{2.4 + j3.2} = \frac{600 \angle 0^\circ}{4 \angle 53.16^\circ}$$

$$= 150 \angle -53.16^\circ \text{ A}$$

$$S_2 = (600 \angle 0^\circ)(150 \angle -53.16^\circ)$$

$$= 90 \angle -53.16^\circ \text{ KVA}$$

$$= 54 + j72 \text{ KVA}$$

2 points

load 3

$$P_3 = 80 \text{ kW}$$

$$\text{pf}_3 = 0.8 \rightarrow \theta_3 = 36.9^\circ$$

$$Q_3 = P_3 \tan \theta_3 = 80 \times 0.75 = 60 \text{ KVAR}$$

$$S_3 = 80 + j60 \text{ KVAR}$$

2 points

$$S_T = P_T + jQ_T$$

$$= (12 + 54 + 80) + j(0 + 72 + 60)$$

$$= 144 + j132 \text{ KVA}$$

$$= 195.35 \angle 42.53^\circ \text{ KVA}$$

b) 3 points

$$\text{pf}_T = \cos 42.53 = 0.737$$

$$\text{pf}_{\text{new}} = 0.9 \rightarrow \theta_{\text{new}} = 25.85^\circ$$

$$Q_{\text{new}} = P_{\text{old}} \tan \theta_{\text{new}} = 144 \tan 25.85 = 69.792 \text{ KVAR}$$

$$Q_{\text{cap}} = Q_{\text{new}} - Q_{\text{old}} = -62.257 \text{ KVAR}$$

$$Q_c = \frac{-V^2}{\frac{1}{\omega C}} \Rightarrow Q_c = -\omega C V^2$$

$$C = \frac{Q_c}{\omega V^2} = \frac{62.257}{2\pi \cdot 50 \times 600^2} = 550.8 \mu\text{F}$$

$$\begin{aligned} \text{c) } I_s^* &= \frac{S_{\text{load}}}{V_s} = \frac{144 + j69.742}{600 \angle 0^\circ} \text{ kVA} \\ &= 240 + j116.24 \text{ A} \end{aligned}$$

$$I_s = 240 - j116.24 \text{ A}$$

3 points

$$S_s = 144 + j69.742 \text{ kVA} = 185.06 \angle 38.4^\circ \text{ kVA}$$

$$\text{d) } S_T = (144 + 102) + j132 = 279.18 \angle 28.23^\circ \text{ A}$$

$$P_{f_T} = \cos(28.23^\circ) = 0.881 < 0.9$$

still a cap is needed

3 points

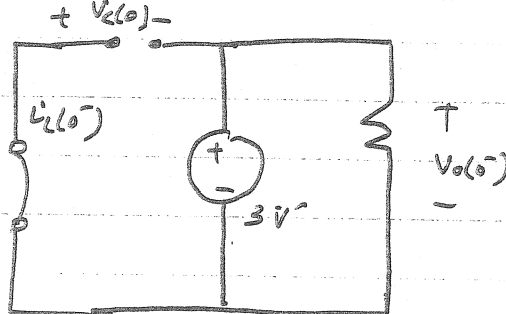
20 points

Q5. a)

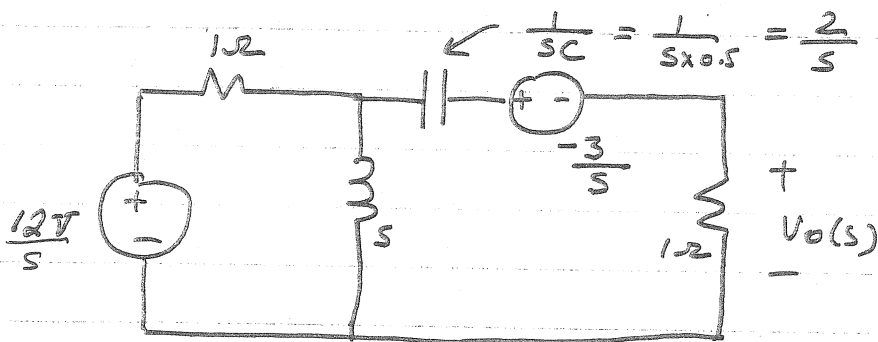
$t = 0^-$  (sw1 open, sw2 closed)

$i_L(0^-) = 0$

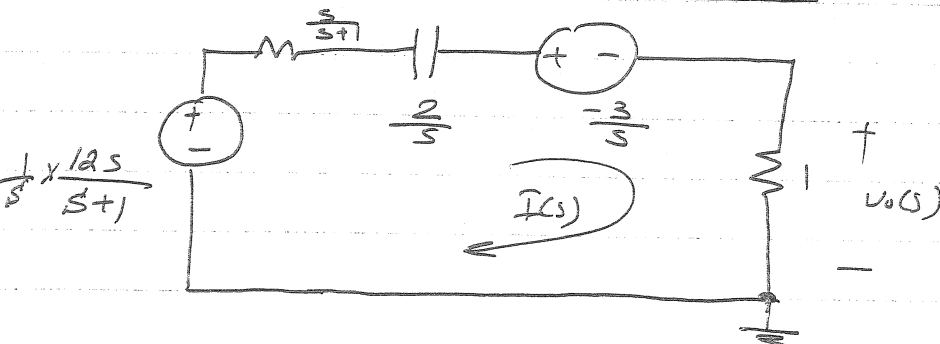
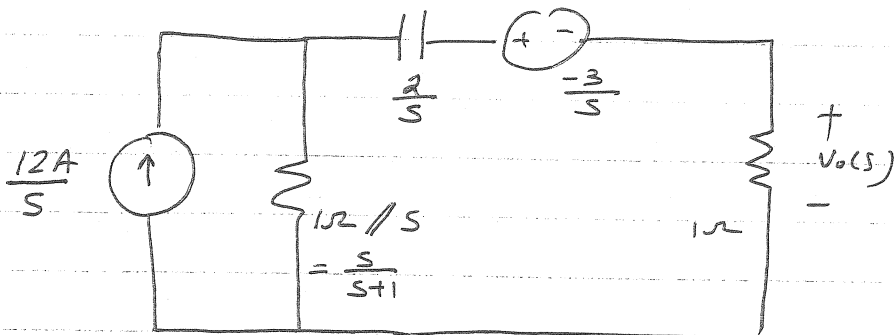
$V_C(0^-) = -3V$



$t = 0^+$



5 points



5 points

$$V_O(s) = I(s) = \left( \frac{12}{s+1} + \frac{3}{s} \right) / \left( 1 + \frac{2}{s} + \frac{s}{s+1} \right)$$

$$= \frac{12s + 3(s+1)}{s(s+1)} \times \frac{s(s+1)}{s(s+1) + 2(s+1) + s^2}$$

$$V_o(s) = \frac{12s + 3s + 1}{2s^2 + 3s + 2} = \frac{15s + 1}{2s^2 + 3s + 2}$$

$$b) V_o(s) = \frac{12(s+3)}{s(s^2+4s+5)} ; \quad s^2+4s+5 = (s+2-j1)(s+2+j1)$$

$$V_o(s) = \frac{12(s+3)}{s(s+2-j1)(s+2+j1)}$$

$$V_o(s) = \frac{K_0}{s} + \frac{K_1}{(s+2-j1)} + \frac{K_1^*}{(s+2+j1)}$$

3 points

$$K_0 = 36/5$$

$$K_1 = (s+2-j1)V_o(s) \Big|_{s=-2+j1} = 3.79 \angle 161.57^\circ$$

$$K_1^* =$$

$$V_o(t) = K_0 + 2|K_1| e^{-2t} \cos(t + 161.57^\circ) \quad \text{V}$$

2 points

$$c) V_o(t) = V_m |H(j\omega)| \cos(\omega t + \angle H(j\omega) + \theta)$$

$$H(j\omega) = \frac{10^3}{(j50)^2 + 2 \times j50 + 10^3} = \frac{10^3}{-2500 + j100 + 1000}$$

$$= \frac{1000}{-1500 + j100} = 0.665 \angle 3.816^\circ$$

5 points

$$V_o(t) = 6.65 \cos(50t + 18.816^\circ) \quad \text{V}$$

Q6. 10 points

Summary

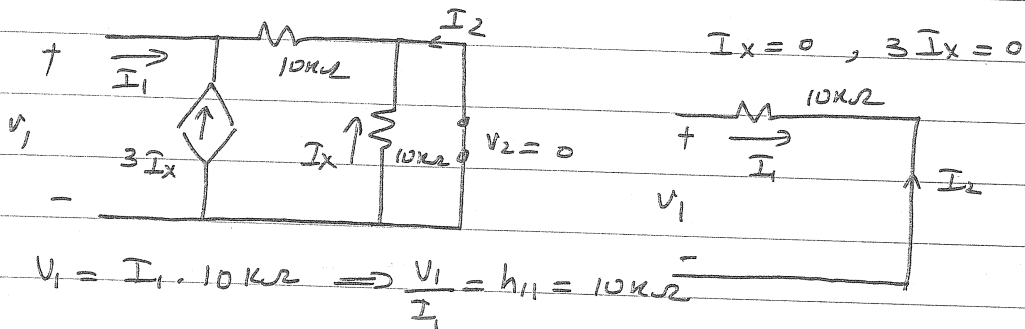
$$h_{11} = \frac{V_1}{I_1} \Big|_{V_2=0} = 10 \text{ k}\Omega$$

$$h_{12} = \frac{V_1}{V_2} \Big|_{I_1=0} = 2$$

$$h_{21} = \frac{I_2}{I_1} \Big|_{V_2=0} = -1$$

$$h_{22} = \frac{I_2}{V_2} \Big|_{I_1=0} = 0.4 \text{ mS}$$

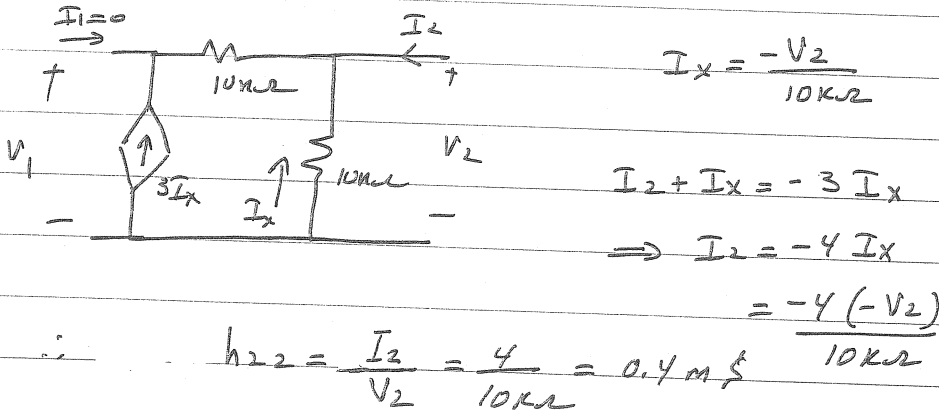
test 1  $V_2 = 0$



$$V_1 = I_1 \cdot 10 \text{ k}\Omega \Rightarrow \frac{V_1}{I_1} = h_{11} = 10 \text{ k}\Omega$$

$$I_2 = -I_1 \Rightarrow \frac{I_2}{I_1} = -1 = h_{21}$$

test 2  $I_1 = 0$



$$\therefore h_{22} = \frac{I_2}{V_2} = \frac{4}{10 \text{ k}\Omega} = 0.4 \text{ mS}$$

$$V_2 = \frac{10 \text{ k}\Omega}{20 \text{ k}\Omega} \cdot V_1 \Rightarrow h_{12} = \frac{V_1}{V_2} = 2$$

10 points

Q7. a)  $\omega = 0$   $V_o = 0$   
(L short, C-open)

2 points  $\omega = \infty$   $V_o = 0$   
(L open, C-short)

$\therefore$  This is Band-pass filter

b)  $\omega_0 = \frac{1}{\sqrt{LC}}$

2 points  $\therefore L = \frac{1}{\omega_0^2 C} = \frac{1}{(314 \cdot 10^3)^2 \cdot 47 \cdot 10^{-9}} = 815.8 \text{ mH}$

c)  $\beta = \frac{1}{RC} = \frac{\omega_0}{Q} = \frac{314 \text{ krad/sec}}{10} = 31.4 \text{ krad/sec}$

2 points

$\therefore R = \frac{1}{\beta \cdot C} = \frac{1}{31.4 \cdot 10^3 \times 47 \cdot 10^{-9}} = 677.6 \text{ } \Omega$

d)  $\omega_{c1,2} = -\beta/2 \pm \sqrt{(\beta/2)^2 + \omega_0^2}$

$\omega_{c1} = 298,692 \text{ krad/sec}$

$\omega_{c2} = 314,392 \text{ krad/sec}$

e) with  $R_2 = 1 \text{ k}\Omega$

$R_{eq} = 1 \text{ k}\Omega \parallel 677.6 \approx 404 \text{ } \Omega$

2 points

$\beta_{new} = \frac{1}{R_{eq} \cdot C} = 52676 \text{ rad/sec}$

$\downarrow$   
higher than before